AGaFe Conference

in honour of Philippe Ellia on his 60^{th} birthday

Abstracts

Ciro Ciliberto: *On Cremona geometry of plane curves.*

(joint work with A. Calabri) A Cremona transformation is a birational self–map of $\mathbb{P}^r$. Cremona transformations of $\mathbb{P}^r$ form a group. It is a classical subject to look at the orbit of a given (irreducible or not) subvariety V of $\mathbb{P}^r$ up to the Cremona group. In this orbit there are elements of minimal degree, called *Cremona minimal*, which are somehow the simplest Cremona representatives of (the elements of the orbit of) V. In this paper I will discuss this concept, with particular attention to the case of plane curves. I will illustrate old and new results on this subject, as well as some open problems.

Edoardo Sernesi : *Syzygies of special line bundles on curves.*

(Jt. work with M. Aprodu) I will discuss geometric conditions called (Δ_q) on a special very ample line bundle L on a projective curve C. I will show that condition (Δ_3) implies that L has the well known property (M_3), generalizing a similar result proved by Voisin for $L = K$.

Alessandro Verra: *Geometry of moduli of Nikulin surfaces in low genus.*

The talk focuses on Nikulin surfaces with a low genus polarization, their moduli and related topics. In low genus at least, the families of Nikulin surfaces to be considered sit in a system of relations to other interesting families of geometric objects. Using them, explicit descriptions of the corresponding moduli spaces, and results on their birational geometry, are possible. The genus 8 case will be specially considered and (uni)rationality results will be proved.

Alessandra Sarti: *Pell's equation and automorphisms of $Hilb^2(K3)$.*

I will present recent results on the automorphism group of the Hilbert scheme of two points on a generic K3 surface of any polarization. In this case the Picard number of the Hilbert scheme is two, which is the minimal possible. In particular by using ampleness results of Bayer-Macri and a detailed study of the isometries of the Picard lattice, I show the existence of non-natural non-symplectic involutions on some Hilbert scheme, depending on the degree of the polarization. In all the results the solutions of certain Pell's equation play a fundamental role. This is joint work with S. Boissire, A. Cattaneo, M. Nieper-Wisskirchen.

Beppe Pareschi: *Bimeromorphic characterization of complex tori.*

The basic results about generic vanishing on smooth complex projective varieties can be extended to the setting of compact Kahler manifolds. As an application, it follows that complex tori are (bimeromorphically) characterized among compact Kahler manifolds by the conditions that the first two plurigenera are equal to one, and that the irregularity is equal to the dimension. This generalizes a result of Chen and Hacon in the projective case. Joint work with Mihnea Popa and Christian Schnell.

Enrico Schlesinger: *Smooth curves specialize to extremal curves.*

We consider locally Cohen-Macaulay curves, that is, curves without isolated or embedded points, in projective 3-space. We show that any smooth curve belongs to an irreducible family containing an extremal curve; we actually find necessary and sufficient conditions for an irreducible component of the Hilbert scheme to contain an extremal curve. Since the extremal curves for a given degree and genus form an irreducible subset of the Hilbert scheme, we conclude that all smooth curves of degree d and genus g belong to the same connected component of the Hilbert scheme of locally Cohen-Macaulay curves. Joint work with Robin Hartshorne and Paolo Lella.

Davide Franco: *Nron-Severi groups of a general hypersurface.*

A well known result of Angelo Lopez allows to compute the Picard group of a general space surface containing a smooth curve. In the talk I report on a recent generalization of Lopez's recipe to the computation of the intermediate Nron-Severi group of a general hypersurface in any smooth variety. In particular I show how such a problem leads very naturally to a "Lefschetz like" question for the Gysin map between homology groups.

Gianfranco Casnati: *Rank two aCM bundles on varieties of dimension at least three.*

In the talk we first give a quick overview on the known result about indecomposable vector bundles of rank two with vanishing intermediate cohomology on an aCM smooth variety of dimension at least three in the projective space.

In the second part of the talk we deal with some new result about the complete classification of such bundles on del Pezzo varieties of dimension at least three.

Robin Hartshorne: *Local cohomology groups as D-modules.*

Local cohomology groups of a ring with respect to an ideal are rarely finitely generated. In special cases they may be cofinite (i.e. descending chain condition), but that is also rare. However, if one regards a local cohomology group as a module over the ring of differential operators (that is, as a D-module), then they become not only finitely generated but actually finite length as D-modules. This allows one to prove many good properties, with applications, for example, to subvarieties of projective varieties.

Paltin Ionescu: *Some special Fano manifolds*

We combine projective geometry with a few Mori techniques on several remarkable embedded manifolds. Some theorems and conjectures are presented, involving prime Fano manifolds which may be: secant or dual defective, connected by conics, of small codimension, of high index, defined by quadratic equations a.s.o. A survey of joint work with Francesco Russo, including some material not yet published.

Alex Massarenti: *On the automorphisms of moduli spaces of curves and stable maps*

The Kontsevich moduli space $\overline{M}_{g,n}(\mathbb{P}^N,d)$ parametrizing stable maps from n-pointed genus g curves to a projective space plays a central role in both algebraic geometry and Gromov-Witten theory.
All automorphisms of these spaces tend to be modular, in the sense that they can be described in terms of the objects parametrized by the moduli space itself.
We will explore this modular behavior of automorphisms in two cases:

- $\overline{M}_{g,n}(\mathbb{P}^0,0) \cong \overline{M}_{g,n}$, the Deligne-Mumford compactification of the moduli spaces of n-pointed, genus g curves. In the case $g=0$ we will study the problem in positive characteristic as well.
- $\overline{M}_{0,n}(\mathbb{P}^{n-3},n-3)$, the space parametrizing degree $n-3$ rational normal curves in $\mathbb{P}^{n-3}$.

Fabrizio Catanese: *(Uni-)Rationality of Ueno-type manifolds and complex dynamics*

Let E be either the anharmonic elliptic curve E_i, which admits complex multiplication by i, the 4-th root of 1, or the equianharmonic (or Fermat) elliptic curve E', which admits complex multiplication by η, the 6-th root of 1.

The Ueno-type manifolds are the minimal resolutions of singularities $X^n_m$of the quotient of E^n by the diagonal action of a cyclic group of order m= 4, or 6 acting with a fixed point. These manifolds are well known to be rational in dimension $n=1,2$. For $n \geq 3$ Ueno around 1973 showed that X^n_4 has Kodaira dimension 0 for $n \geq 4$, X^n_6 has Kodaira dimension 0 for $n \geq 6$, and asked whether X^3_4, and X^n_6 $(3 \leq n \leq 5)$ are unirational.

Interest for this open questions was revived on one side by Campana, who showed that X^3_4 is rationally connected, and hoped that it is not unirational: whence the name Campana-Ueno manifold was proposed for X^3_4.

The rebirth of interest in the rationality of these manifolds stems also from complex dynamics and entropy, since these manifolds admit an action by $GL(n,\mathbb{Z})$. Oguiso and Truong proved the rationality of Y^3_6, and showed that in this way one gets a rational variety with a primitive automorphism of positive entropy.

Unirationality of X^3_4 was proven in a joint work with Oguiso and Truong, later Colliot Thelene showed, using our conic bundle realization, that X^3_4 is indeed rational (even if the conic bundle is a non trivial element of the Brauer group).

Together with Oguiso, we proved the unirationality of X^4_6, showing that it is birational to a diagonal cubic surface over the function field $\mathbb{C}(x,y)$ admitting 27 rational points. Using classical results of Segre, Swinnerton-Dyer and Colliot Thelene we show that the surface is unirational, but it is not rational. Is it possible, like done for the conic bundle case, to change the cubic surface birationally and prove rationality of X^4_6?

The unirationality of X^5_6 is also an open question.

Rosa M. Miró-Roig: *Monomial Togliatti systems of cubics.*

The goal of my talk is to establish a close relationship between a priori two unrelated problems: (1) the existence of homogeneous artinian ideals $I \subset k[x_0,\dots,x_n]$ which fail the Weak Lefschetz Property; and (2) the existence of (smooth) projective varieties $X \subset \mathbb{P}^N$ satisfying at least one Laplace equation of order $s \geq 2$. These are two longstanding problems which lie at the crossroads between Commutative Algebra, Algebraic Geometry, Differential Geometry and Combinatorics. In the toric case, I will classify some relevant examples and as byproduct I will provide counterexamples to Ilardis conjecture. Finally, I will classify all smooth Togliatti system of cubics and solve a conjecture stated in my joint work with Mezzetti and Ottaviani.

All I will say is based in joint work with either E. Mezzetti and G. Ottaviani or M. Michalek.

Emilia Mezzetti: *Skew-symmetric matrices and globally generated vector bundles.*

I will speak on recent joint work with Ada Boralevi, about the problem of determining all pairs (c_1, c_2) of Chern classes of rank 2 bundles that are cokernel of a skew-symmetric matrix of linear forms in 3 variables, having constant rank$2c_1$ and size $2c_1 + 2$. We completely solve the problem in the "stable" range, i.e. for pairs with $c_1^2 - 4c_2 < 0$, proving that the additional condition $c_2 \leq \binom{c_1+1}{2}$ is necessary and sufficient. For $c_1^2 - 4c_2 \geq 0$, we prove that there exist globally generated bundles, some even defining an embedding of $\mathbb{P}^2$ in a Grassmannian, that cannot correspond to a matrix of the above type. This extends previous work on $c_1 \leq 3$.

Christian Peskine: *Order one congruences of lines with smooth fundamental locus*

A *congruence of lines* in $\mathbb{P}^N$ is an $(N-1)$-dimensional subvariety of the Grassmann variety. The *order* $o(\Sigma)$ of a congruence $\Sigma \subset G(1, N)$ is the number of its lines passing through a general point of $\mathbb{P}^N$. The *fundamental locus* $X(\Sigma)$ of the congruence is the set of points of $\mathbb{P}^N$ through which pass infinitely many lines of Σ. When $o(\Sigma) = 1$, the fundamental locus has a natural scheme structure. As an example, we see that the 2-secant lines to a twisted cubic curve form a congruence of order 1 whose fundamental locus is set-theoretically but not scheme-theoretically the twisted cubic.

We study smooth linear congruences of order 1 with smooth fundamental locus. The lines of such a congruence Σ are the k-secants to $X(\Sigma)$ with $k = (N-1)/(N - dim(X(\Sigma)) - 1)$. We define k as the secant index of Σ. Furthermore $X(\Sigma)$ is equipped with a ramification divisor cut out by a virtual hypersurface of degree $k-2$. We note also that the congruence itself is a Fano variety of index $(N - dim(X(\Sigma)))$.

All congruences of order 1 with smooth fundamental locus and with secant index 1 or 2 are described by elementary means. We note (this is the origin of this work) that there exist precisely four families with secant index 3 (there fundamental loci are the celebrated projected Severi Varieties) and that two families with secant index 4 are known (there fundamental loci are Palatini varieties of dimension 3 and 6). We do not know any congruence of order one with smooth fundamental locus and secant index ≥ 5.

As a conclusion we note with amusement that any new congruence of order one with smooth fundamental locus would be a counter-example to the celebrated conjecture of Hartshorne concerning low codimension varieties.